The End of the Integral Calculus Illusion

Every closed form passes through its centre: the fold around 4.5, the chain ½ × ⅔, and the folded bar

Filipe Bastos
ARC corpus (Architecture of Relational Coherence)
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> *Stop counting grains of sand to weigh the beach. Weigh the beach.*

How to read this version. Version 3.0 (March 2026) was one of the author's first discoveries: the passage from surface to volume is not an accumulation of slices but a fold around the barycentre 4.5, at a cost of ½. Part I restates that discovery and marks the four places where the corpus has since corrected it; under the corpus rule, every retired line is shown with its replacement. Part II adds what was established in the night of 3 to 4 November 2026: the method closes on every closed form once each face takes its own height to the centre, and forms that look open (the U, the horseshoe, the ring) are bars folded around their centre line.

Statuses follow the corpus. **Th**: a finite computation, checked by the script `integral_v4_verif.py` distributed with this piece (34 checks, all passed). **F**: a fact of received science, cited within its original scope. **L**: a reading, declared as such. A Th cannot be overturned by a reading, and a reading does not borrow the force of a Th.

Part I. The March discovery

1. The impasse of the continuum

L. Since Newton and Leibniz, a whole is measured by summing an endless number of vanishing parts. The integral does not know the volume; it rebuilds it by accumulation. **The Completeness Principle** (v3, Axiom 1): an operation that produces a real quantity (a volume, an energy) should resolve to a whole or an exact ratio at every step. An operation that needs endlessly many incomplete parts to approach the whole confuses the limit with the thing. The volume was already there.

2. The multiplication tables switch at the centre

Write root(n) for the digit sum of n, repeated until one digit remains, with 9 staying 9.

×m	k = 1 2 3 4	k = 5 6 7 8	k = 9
×1	1 2 3 4	5 6 7 8	9
×2	2 4 6 8	1 3 5 7	9
×3	3 6 9 3	6 9 3 6	9
×4	4 8 3 7	2 6 1 5	9
×5	5 1 6 2	7 3 8 4	9
×6	6 3 9 6	3 9 6 3	9
×7	7 5 3 1	8 6 4 2	9
×8	8 7 6 5	4 3 2 1	9

Th (the switch 4 | 5). In every table ×1 to ×8, position k and position 9 − k close on 9. The fold of each table lies between positions 4 and 5, at 4.5.

Th (multiplicative completeness). For every pair (m, 9 − m) and every k: root(mk) + root((9 − m)k) closes on 9, because mk + (9 − m)k = 9k. The tables work in inversion pairs.

3. The pivot: surface becomes volume by a fold

Th (closure by the pivot, v3 Th 3.1). Turn the segment 1 to 8 around a pivot p. Each digit x sweeps a radius |x − p| and its partner 9 − x a radius |9 − x − p|. The two ends of every pair meet on one sphere only when p = 4.5. For p = 5 or p = 4 the form stays open.

L. The fold is one act, a turn around 4.5. Its cost is the ratio of the folded depth to the unfolded extent: 3.5 / 7 = ½ for the outer pair (1, 8). Integration tries to rebuild this fold with endlessly many flat slices.

4. π as 11 / 3.5

Th. 11 / 3.5 = 22 / 7 exactly.
L. 11 is the interface, the first step past the closure 9; 3.5 is the half-span of the pair (1, 8), the radius of the fold. Writing 22/7 multiplies the half away and hides why 22 and why 7. The corpus holds 22/7 as the structural address of the circle, not as its measured value; Lindemann's theorem stands (v3, Remark 4.1).

5. The barycentre 4.5

Th. 4.5 is the only fixed point of x ↦ 9 − x, and the midpoint of every pair (1, 8), (2, 7), (3, 6), (4, 5). Hence k = 4.5 / 9 = ½.

6. The seven convergences to ½

Path	Type	Derivation	Result
I	global	4.5 / 9	½
II	local	3.5 / 7	½
III	geometric	11 / (22/7) = 3.5, then 3.5 / 7	½
IV	all pairs	half-span over span	½
V	sphere	expected value of |cos θ| on the sphere	½
VI	tables	switch at position 4.5 of 9	½
VII	physical	spin ½	½

Th for each equality (III under the address 22/7). **L** for their joint reading. The corpus examined their independence (Volume 2, chapter 11): I, II, IV and VI are the same identity, a half-span over its span; V is an independent computation and the strongest of the seven; VII is a reading of a physical fact. The robust convergence is that of three distinct objects: the arithmetic fold around 4.5, the geometry of the two circles, and the projection onto the sphere.

7. The four corrections to version 3.0

C1. The volume formula. Retired: "V = S × k, k = 0.5, replaces all integration for coherent systems" (v3, abstract and §9), checked on the sphere alone for eighteen months. Replaced by: **face × ½ × ⅔ × h**, with h the height from the centre to that face (September 2026 errata; corpus rule R11: a general result is tested on a second example from another family before it is inscribed). The v3 sphere check, $4\pi R^2 \times ½ \times 2R/3$, already carried the ⅔.

C2. The reach of the formula. Retired: the corpus delimitation "ARC does not claim to abolish the integral universally" with Conjecture 9.2 left open. Replaced by: Theorems A and B of Part II, which close the conjecture on the scope stated in section 14.

C3. Path VII. Retired: the reading of the 720° return of the fermion as two passes through the hemispheres (v3, §6 and §7). The 1975 measurement (the phase sign that changes after a 360° precession) remains a fact; its reading as a double rotation is withdrawn (Volume 3, chapter 13). Replaced by: spin as the pivot between 4 and 5, the half turn that inverts and binds; ½ remains the cost of deciding which side of 4.5 one stands on.

C4. Reference [4]. The v3 reference "A Proof of the Riemann Hypothesis" is kept as a dated title only. The corpus now holds that the question of Riemann has no object in the register of the sum and the product (*Le Fini et l'Infini mêlés*, v1.2).

Part II. 4 November 2026: every closed form passes through its centre

8. The chain of the half

Th. 9 × ½ = 4.5; 4.5 × ⅔ = 3; 3 / 9 = ⅓; and ½ × ⅔ = ⅓.
L. The author's 4.5 is the half of 9, and it is split into three. Two thirds of it is 3, and 3 on 9 is

the third of the pyramid. The third is reached by passing through the half: the ½ is the distance from the centre to the surface, the ⅔ is the bridge, their product is the point. The cone, and every closed volume below, is read along this one chain.

9. Theorem A: a closed form is computed from its centre

Th A. Let a closed form have flat faces and let c be a point inside it that sees every face. Each face, joined to c, is the base of a cone. Then

V = Σ over the faces of (face × ½ × ⅔ × h),

where h is the height from c to the plane of that face. Every height is positive, and the cones fill the form exactly once. The total is the same for every such point c.

Why. A cone of base B and height h has volume B × h / 3 (Eudoxus, Archimedes), and B × h / 3 = B × ½ × ⅔ × h. When c sees every face, the cones from c meet only along their sides and together make the whole form.

Each face takes its own height. A single height for every face works only when one sphere touches every face (sphere, cube, regular pyramid, cone). On a box 1 × 2 × 5 the single height ½ gives 34 × ⅓ × ½ = 17/3 instead of 10; on a box 1 × 1 × 2 it gives 5/3 instead of 2. The method is not at fault: one height was given to six faces that each have their own.

Form	Centre	Result	Exact value
Box 1 × 2 × 5	any interior point	10	10
Irregular tetrahedron	its centre	10	10
Five random convex polyhedra	their centre	equal to each hull volume	all heights positive
L prism (a hollowed form)	its centre (5/6, 5/6, ½)	3	3
Sphere of radius R	its centre	$4\pi R^2 \times ½ \times 2R/3$	$4/3\ \pi R^3$

L. The point c is the centre of least tension, the 4.5 of the form. In a convex form it always lies inside and sees every face. Everything passes through the centre.

10. The form comes before the measure

Th (plane). In the plane the same gesture reads side × ½ × height to the centre. Rectangle 4 × 3 from its centre: ½ × (4 × 1.5 × 2 + 3 × 2 × 2) = 12. Its cross and its two diagonals cut it into 8 triangles of equal area 1.5, and 8 × 1.5 = 12.

Th (stretch). In the square the same 8 triangles have 45° angles. The rectangle carries them unfolded: the angles open, the 8 equal triangles from the centre remain. Stretching keeps the form and the proportions, not the angles. The ellipsoid of half-axes 1, 2, 3 is the sphere of radius 1 stretched by 1, 2 and 3: its volume is 4/3 π × 6, exact, with no slices.

L. It is the form, not the measure of the form.

11. Theorem B: the folded bar

Th B. A bar of section A folded along its centre line keeps its content: V = A × length of the centre line. At every fold the inner side is shortened and the outer side is lengthened by exactly the same amount; the centre line keeps its length.

Form	Centre line	Result	Count
U of the 3 × 3 square, thickness 1	2.5 + 2 + 2.5 = 7	7	7 cells
U 4 × 3	2.5 + 3 + 2.5 = 8	8	8 cells
U 5 × 3	9	9	9 cells
U 4 × 4	10	10	10 cells
Torus, R = 3, r = 1	2πR	πr² × 2πR = 2π²Rr²	

Th (the two halves of the torus). The inner half is $\pi^2Rr^2 - 4/3\ \pi r^3$ and the outer half is $\pi^2Rr^2 + 4/3\ \pi r^3$. They differ from the centre line by exactly $4/3\ \pi r^3$ each, the volume of a sphere of radius r, and together they give $2\pi^2Rr^2$. (The received integral was run once, as an outside control; it agrees.)

L. The U is a bar. Its void counts for nothing and enters no computation. The one has three points (inner edge, centre, outer edge) and two parts, and the two parts are a pair: what one loses, the other carries, and the centre does not move. This is the v3 pivot again, r(x) = r(9 − x), read on a bar. The ring is the same bar, closed.

F. Workshops have long computed bent parts this way. In sheet-metal and bar bending, the developed length is taken along the **neutral fibre**, the line in the thickness that is neither stretched nor compressed. Its place in the thickness is the **K factor**, about 0.5 for a wide bend and down to about 0.3 for a tight one, depending on the material and the bend radius.

L. On the rule of 9, K = ½ is the fibre at 4.5, the centre; K near ⅓ is the fibre at 3, two thirds of 4.5. Bending stays between the centre and its two thirds: the chain of section 8 draws its bounds.

12. How received mathematics reads the U

Th. Received geometry classes the U as a non-convex polygon or polyhedron. Its area is computed either by cutting it into rectangles or by the shoelace formula, also called **Gauss's area formula** (Meister 1769, Gauss 1795), which runs round the vertices and counts some triangles with a minus sign. The centroid of the U falls outside its matter, in the opening. Topology, on the other hand, holds the U, the rectangle and the disc to be the same form (one is deformed into the other without cutting or gluing), while the torus has a true hole.

L. Each branch keeps one half. Topology keeps the identity of the U with the rectangle and throws away the measure; geometry keeps the measure and throws away the identity. The fold keeps both: the U is a bar, read from its centre line, with no signed area. The author reads the U as the square of Gauss, the square with its closure missing; the signed areas are the price of the closure withdrawn, as 9 set equal to 0.

13. Gnomons: the 7 and the 37

Th. $4^2 - 3^2 = 7 = 4 + 3$, and $n^2 - (n-1)^2 = 2n - 1$: each passage from one square to the next adds an odd gnomon. $4^3 - 3^3 = 37 = 16 + 12 + 9 = 3 \times 4 \times 3 + 1$, a centred hexagonal number, root 1; and $n^3 - (n-1)^3 = 3n(n-1) + 1$. The U of the 3×3 square is $9 - 2 = 7$.

L. The square returns the 7 (the 2 opened, the pair (2, 7)); the cube returns the 1 and its partner 8 (the U 4×3 has content 8, and $1 + 8 = 9$). The author reads the U of the 3×3 square as the native form of the one blown into a two.

14. The scope, and Conjecture 9.2

Conjecture 9.2 (*Why This Is Not Numerology*, v2.9, §9) held that every volume of physical interest decomposes into a finite sum of rotational primitives, and was left open.

Th. Theorem A closes every form with finitely many flat faces, convex or hollowed, from a centre that sees every face. The same gesture closes the curved forms born of a turn, a stretch or a fold: the sphere, the ellipsoid, the solids of revolution, the folded bar and the torus (Theorem B and section 10). On this scope, Conjecture 9.2 is closed.

What stays outside the Th. A curved surface that is born of no turn, stretch or fold (a stone, say) has no finite list of faces, and section 9 needs one. **L.** Matter is finite: a real body is a finite assembly, and its surface has finitely many faces. Read this way, Theorem A applies to every real body; the endless slice belongs to the instrument, not to the body. This last step is a reading and is marked as one.

What the piece does not say. It does not dispute any number that the integral produces: on every form above, the two methods agree. It disputes the necessity of the detour through endless slices for a closed form: the content of the form is reached from its centre in a finite number of steps.

Appendix. Verification

The script `integral_v4_verif.py` (Python 3, exact fractions for every polyhedral case) runs 34 checks, all passed: the chain of the half (4); the tables (2); closed polyhedra from the centre, including the single-height counterexamples (11); plane forms (3); sphere, ellipsoid and torus by closed forms, with one outside control by the received integral (5); the U as a folded bar (5); the gnomons (4).

Sources

F. Bastos, *The End of the Integral Calculus Illusion*, v3.0, Seven Convergences Edition, March 2026. F. Bastos, *Why This Is Not Numerology*, v2.9, §9 (Conjecture 9.2). ARC corpus, Volume 2, *La Figure*, chapter 11; Volume 3, *Le Code et le Registre*, chapters 3, 8, 13 and 17 (corrections C1 to C3, rule R11). F. Bastos, *Le Fini et l'Infini mêlés*, v1.2. Archimedes, *The Method*. Pappus of Alexandria, *Collection*, book VII (the centroid theorem). A. L. F. Meister (1769) and C. F. Gauss (1795) on the shoelace area formula.